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WITH THE ASSISTANCE OF

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THE SUMMER MEETING IN STATE COLLEGE

The forty-third Summer Meeting of the Society and the twentieth Colloquium were held at The Pennsylvania State College, State College, Pennsylvania, from Tuesday to Friday, September 7–10, 1937. The Mathematical Association of America met Monday afternoon and Tuesday morning. About four hundred seventy-five persons attended the meetings, among whom were the following two hundred seventy-four members of the Society:

C. R. Adams, R. B. Adams, V. W. Adkisson, R. P. Agnew, L. V. Ahlfors, H. W. Alexander, E. B. Allen, V. A. Ames, R. L. Anderson, R. C. Archibald, H. T. R. Aude, Frank Ayres, W. L. Ayres, C. L. Bacon, H. M. Bacon, Reinhold Baer, I. A. Barnett, E. R. Beckwith, Garrett Birkhoff, G. D. Birkhoff, A. H. Black, H. L. Black, H. F. Blichfeldt, G. A. Bliss, Henry Blumberg, R. P. Boas, H. R. Brahana, C. C. Bramble, F. L. Brooks, E. T. Browne, J. A. Bullard, F. J. H. Burkett, J. H. Bushey, J. Hughes Bushey, W. H. Bussey, W. D. Cairns, J. W. Calkin, C. C. Camp, W. B. Campbell, Leonard Carlitz, I. S. Carroll, E. W. Chittenden, R. V. Churchill, A. B. Coble, Teresa Cohen, J. B. Coleman, L. P. Copeland, Byron Cosby, Richard Courant, H. S. M. Coxeter, H. B. Curry, D. R. Curtiss, J. H. Curtiss, E. H. Cutler, Tobias Dantzig, P. H. Daus, H. A. Davis, H. T. Davis, J. E. Davis, F. F. Decker, L. L. Dines, C. H. Dix, J. L. Doob, H. L. Dorwart, R. H. Downing, Arnold Dresden, D. M. Dribin, J. A. Eiesland, O. J. Farrell, E. J. Finan, M. M. Flood, W. B. Ford, Tomlinson Fort, R. M. Foster, J. S. Frame, Bernard Friedman, Kurt Friedrichs, A. H. Frink, Orrin Frink, T. C. Fry, A. S. Gale, B. P. Gill, R. E. Gilman, J. W. Givens, W. O. Gordon, W. C. Graustein, T. E. Gravatt, C. H. Graves, W. L. Graves, M. B. Greenebaum, J. A. Greenwood, F. L. Griffin, V. G. Grove, Margaret Gurney, B. L. Hagen, Israel Halperin, J. A. Hamilton, E. H. Hanson, W. L. Hart, M. L. Hartung, Alan Hazeltine, G. A. Hedlund, E. R. Hedrick, Coleman Herpel, H. C. Hicks, E. H. C. Hildebrandt, T. H. Hildebrandt, T. R. Hollcroft, Charles Hopkins, W. A. Hurwitz, W. L. Hutchings, M. H. Ingraham, Dunham Jackson, Nathan Jacobson, R. L. Jeffrey, F. A. Jeffries, Fritz John, Evan Johnson, R. A. Johnson, R. F. Johnson, B. W. Jones, H. S. Kaltenborn, A. J. Kempner, L. S. Kennison, G. S. Ketchum, P. W. Ketchum, S. C. Kleene, J. R. Kline, M. S. Knebelman, J. C. Knipp, H. L. Krall, C. C. Krieger, W. D. Lambert, K. W. Lamson, O. E. Lancaster, A. E. Landry, E. P. Lane, R. E. Langer, C. G. Latimer, V. V. Latshaw, Solomon Lefschetz, D. H. Lehmer, D. C. Lewis, F. P. Lewis, Marie Litzinger, T. R. Long, L. L. Lowenstein, R. G. Lubben, N. H. McCoy, W. H. McEwen, R. S. McKee, J. V. McKelvey, J. C. C. McKinsey, L. A. MacColl, C. C. MacDuffee, Saunders MacLane, H. F. MacNeish, A. J. Maria, W. T. Martin, A. E. Meder, W. I. Miller, U. G. Mitchell, E. B. Mode, Virginia Modesitt, E. C. Molina, E. I. Moody, C. N. Moore, R. L. Moore, C. B. Morrey, Richard

Morris, A. P. Morse, H. M. Morse, David Moskovitz, E. J. Moulton, J. R. Musselman, John von Neumann, M. W. Newson, C. O. Oakley, R. E. O'Connor, E. G. Olds, F. W. Owens, H. B. Owens, J. C. Oxtoby, Gordon Pall, H. A. Perkins, R. M. Peters, T. S. Peterson, A. E. Pitcher, H. A. Rademacher, O. J. Ramler, W. C. Randels, J. F. Randolph, Robert Rawhouser, M. S. Rees, C. N. Reynolds, C. E. Rhodes, Moses Richardson, R. G. D. Richardson, D. E. Richmond, H. L. Rietz, J. F. Ritt, H. P. Robertson, G. de B. Robinson, L. B. Robinson, S. L. Robinson, J. B. Rosenbach, C. A. Rupp, N. E. Rutt, A. C. Schaeffer, I. J. Schoenberg, G. E. Schweigert, C. H. W. Sedgewick, R. W. Sedgewick, Wladimir Seidel, I. M. Sheffer, L. W. Sheridan, Max Shiffman, J. A. Shohat, C. G. Shover, R. G. Simond, M. E. Sinclair, James Singer, E. R. Sleight, C. S. Slichter, M. M. Slotnick, L. L. Smail, C. E. Smith, W. M. Smith, Virgil Snyder, G. W. Starcher, O. H. Stecker, N. E. Steenrod, M. H. Stone, M. M. Sullivan, J. L. Synge, Gabriel Szegö, J. D. Tamarkin, A. H. Taub, A. E. Taylor, J. S. Taylor, M. E. Taylor, T. Y. Thomas, C. C. Torrance, W. J. Trjitzinsky, A. W. Tucker, Annita Tuller, B. M. Turner, J. L. Vanderslice, H. S. Vandiver, I. L. Van Dyck, H. E. Vaughan, M. E. G. Waddell, C. C. Wagner, R. W. Wagner, R. J. Walker, J. L. Walsh, R. M. Walter, S. E. Warschawski, Warren Weaver, M. S. Webster, J. V. Wehausen, A. P. Wheeler, A. L. Whiteman, E. A. Whitman, Hassler Whitney, Norbert Wiener, L. R. Wilcox, K. P. Williams, E. W. Wilson, F. E. Wood, F. L. Wren, F. M. Wright, Oscar Zariski, Leo Zippin.

The Colloquium Lectures on the subject *Continuous geometry* were delivered by Professor John von Neumann of The Institute for Advanced Study on Tuesday afternoon and Wednesday, Thursday, and Friday mornings. The presiding officers at the four lectures were, in order, President R. L. Moore, Vice-President Norbert Wiener, Dean G. D. Birkhoff, and Professor M. H. Stone. The attendance was one hundred sixty-nine. These lectures will be published as a volume of the Colloquium Publications.

On Thursday afternoon, by invitation of the Program Committee, Professor Hassler Whitney of Harvard University gave an address entitled *Topological properties of differentiable manifolds*. This address will appear in an early issue of the Bulletin.

President R. L. Moore presided at the business meeting and general session Thursday afternoon; Professor J. R. Kline at the general session Wednesday morning. The presiding officers at the sectional sessions were as follows: Analysis, Professors R. P. Agnew, L. V. Ahlfors, and W. J. Trjitzinsky; Algebra, Mr. Garrett Birkhoff and Professor Gordon Pall; Topology, President R. L. Moore; Geometry, Professor Oscar Zariski.

Headquarters for the meetings were in the lobby of Old Main.

The dormitories of the College were available, at a reasonable charge, to the members of both organizations and their guests.

Arrangements for the entertainment of those attending the meetings were excellent. They began Monday morning with a conducted tour in the neighboring mountains. On Monday evening, a musical recital by Mrs. C. C. Wagner, Professor Teresa Cohen, and Professor T. C. Benton was given at the Nittany Lion Inn. Tea was served in Old Main by the ladies of the local Department of Mathematics on Monday and Tuesday afternoons.

A reception was held Tuesday evening in Old Main at which President Ralph D. Hetzel of The Pennsylvania State College and Mrs. Hetzel greeted the mathematicians and their guests.

On Wednesday, a special luncheon for women, sponsored by the local chapter of Sigma Delta Epsilon, was held in honor of the women who were pioneers in mathematical research in America. The guests of honor were Professor Mary Winston Newson (Ph.D., Göttingen, 1897), the first American woman to receive the Ph.D. degree in mathematics at a German university, and Professors Clara E. Smith and Clara L. Bacon. The attendance was sixty-seven. Wednesday afternoon was devoted to a picnic excursion to Nature Study Camp, Seven Mountains.

On Thursday morning, there was a textile exhibit and a lecture, *Science in the home*, by Dr. Pauline Mack, Professor of Textile Chemistry at The Pennsylvania State College. On Thursday afternoon, Professor and Mrs. F. W. Owens entertained at their home.

A joint dinner for the members of the Society and the Association was held Thursday evening at the Nittany Lion Inn. Professor Virgil Snyder was toastmaster. The first speaker, Dean H. P. Hammond of The Pennsylvania State College, welcomed the mathematicians to State College. The other speakers were Professor R. L. Moore, President of the Society, Professor A. J. Kempner, President of the Association, and Dr. T. C. Fry, Executive Secretary of the Committee on the Semicentennial of the Society. Professor W. L. Hart presented a joint resolution on behalf of the members of both mathematical organizations thanking the President, officers, and members of the Department of Mathematics of The Pennsylvania State College

for their competent arrangements and cordial hospitality. The attendance at the dinner was three hundred eleven.

The Council met on Tuesday, September 7, at 1:00 p.m., again on Wednesday, September 8, at 8:00 p.m., and finally at 10:00 p.m. on September 9, all the sessions being in Old Main, Pennsylvania State College.

The Secretary announced the election of the following persons to membership in the Society:

Mr. Howard Wright Alexander, Lehigh University;
Dr. Philip Osborne Bell, University of Kansas;
Mr. Lafayette Boyd Hedge, Brown University;
Professor Ralph Ernest Huston, Rensselaer Polytechnic Institute;
Professor Henri Albert Jordan, Georgetown University;
Dr. Otis Ewing Lancaster, University of Maryland;
Mr. William Spencer Litterick, The Peddie School, Hightstown, N. J.;
Miss Virginia Modesitt, University of Illinois;
Professor Parry Moon, Massachusetts Institute of Technology;
Mr. Charles W. Moran, Loyola University, Chicago, Ill.;
Mr. Haim Reingold, University of Cincinnati;
Mr. Eugene Kerfoot Ritter, University of Richmond;
Professor Ching-lai Shen, Soochow University, Soochow, China;
Mr. Seymour Sherman, Cornell University;
Professor Felix Perry Welch, Mississippi State College.

As nominees of the California Institute of Technology:
Mr. Hubert Andrew Arnold, California Institute of Technology;
Mr. Robert Palmer Dilworth, California Institute of Technology;
Mr. Newman Arnold Hall, California Institute of Technology;
Mr. Donald Holmes Hyers, California Institute of Technology;
Mr. Edwin Woolman Paxson, California Institute of Technology.

As nominees of Lehigh University:
Mr. Edward Stewart Kennedy, Lehigh University;
Mr. Donald Long Waidelich, Lehigh University.

As a nominee of the Metropolitan Life Insurance Company:
Mr. James A. Hamilton, Metropolitan Life Insurance Company, New York, N. Y.

As a nominee of the Department of Mathematics of Wayne University:
Mr. Anthony Gerard Fleiger, Detroit Edison Company, Detroit, Mich.

It was announced that Dr. Isaac Opatowski, Royal Higher Institute of Engineering, Turin, Italy, had entered the Society under the reciprocity agreement with the Unione Matematica Italiana.

The following institution and department were elected to institutional membership in the Society:

Trinity College, Hartford, Conn.;
Department of Mathematics, Vassar College.

The following appointments by President Moore were reported: as a member of the committee to consider the question of the Society's sponsorship of the William Lowell Putnam Competition to replace Professor J. D. Tamarkin, Professor C. G. Latimer (new chairman, Professor Virgil Snyder); as representative of the Society at the Semicentennial Celebration of the Founding of the University of Wyoming, Professor C. A. Hutchinson; as representative of the Society at the One Hundredth Anniversary of the Founding of Davidson College, Davidson, North Carolina, on June 6–8, Professor J. B. Coleman; as representative of the Society at the Third International American Conference to be held in Mexico City in August, 1937, Professor M. S. Vallarta; as a committee to consider the Society's connection with the American Documentation Institute, Dr. H. W. Tyler (chairman), Professors R. C. Archibald and R. L. Wilder.

Various announcements were made regarding the Semicentennial Celebration of the Founding of the American Mathematical Society, a notice of which appeared on page 593 of the September number of this Bulletin.

The 1938 meeting in Chicago is to be held on April 8–9. The place of the Annual Meeting of 1938 is to be Richmond and Williamsburg.

It was announced that the 1937 Gibbs Lecture is to be presented by Professor C. A. Kraus, his title being *The present status of the theory of electrolytes*.

Committees on the International Congress of Mathematicians to be held in Cambridge, Massachusetts, in 1940 were appointed as follows:

Organizing Committee: Professor W. C. Graustein (chairman), Dean G. D. Birkhoff, Professors G. A. Bliss, G. C. Evans, J. R. Kline, Solomon Lefschetz, and Oystein Ore, Dean R. G. D. Richardson, Professors M. H. Stone, J. L. Synge, J. D. Tamarkin, J. M. Thomas, and Norbert Wiener.

Editorial Committee: Professors Einar Hille (chairman), F. D. Murnaghan, and P. A. Smith.

Financial Committee: Professors Marston Morse (chairman), J. L. Coolidge, M. H. Ingraham, and H. B. Phillips.

Secretary: Dean R. G. D. Richardson.

Two committees on arrangements were appointed: for the Charlottesville Meeting, April 15–16, 1938, Professors G. T. Whyburn (chairman), J. J. Luck, and T. R. Hollcroft; for the Annual Meeting, 1938, Professors J. M. Stetson (chairman), E. J. McShane, G. A. Larew, T. R. Hollcroft, and C. H. Wheeler.

The Council recommended to the Society that the by-laws be amended so as to provide for a fourth Associate Secretary.

Professors A. A. Albert and M. H. Stone were invited to give Colloquium lectures at the Summer Meeting of 1939. A book by Professor C. N. Moore entitled *Summable Series and Convergence Factors* was accepted for the Colloquium Publications.

In view of the decision of the Mathematical Association of America to give the full right of way to the Society at the Semicentennial Celebration and not to hold a meeting itself, a cordial invitation was extended to its members to participate in all festivities.

A resolution regarding the work of Professor H. E. Slaught was adopted as follows:

The Council notes with deep regret the death of Herbert Ellsworth Slaught on May 21, 1937. An active member of the Society almost from its foundation, secretary of the Chicago Section of the Society from 1906 to 1916, and always an enthusiastic and intelligent promoter of mathematics, of mathematical education, and of the interests of mathematicians, he contributed uniquely to the development of mathematics in America. As a teacher and an editor he rendered unusual service, but probably his greatest contribution lay in his success in founding the Mathematical Association of America, devoted to the interests of collegiate mathematics. After the American Mathematical Society in 1915 voted to restrict its activities to the promotion of mathematical research, it was under Professor Slaught's inspiration and guidance that the Association was formed; and it was through his wise counsels that its policies were developed in such a manner that it has so effectively and so harmoniously supplemented the work of the Society. The Council of the Society takes this occasion to express its appreciation of the fine achievements of the Association, and of the uniformly friendly and effective cooperation which has obtained between the Society and the Association since the latter was founded.

The titles and cross references to abstracts of papers read at this meeting are given below. Papers numbered 1 to 11 were presented Tuesday afternoon; papers 12 to 19 Wednesday morning; papers 20 to 40 Thursday morning; paper 41 Thursday afternoon; and papers 42 to 54 Friday morning. The remainder were read by title. Rev. W. C. Doyle was introduced by Professor Francis Regan, Mr. J. K. L. MacDonald by Professor W. A. Hurwitz, Mr. H. W. Alexander by Professor M. S. Knebelman, Mr. C. E. Springer by Professor E. D. Meacham, Mrs. E. M. Torrance by Professor J. D. Tamarkin, Dr. D. L. Netzorg by Professor W. J. Trjitzinsky, Mr. Philip Hall by Mr. Garrett Birkhoff, Mr. V. G. Iyer and Professor O. E. Glenn by Professor T. R. Hollcroft, Mr. J. K. Senior by Professor A. C. Lunn. Paper number 21 was presented by Dr. G. S. Ketchum, 31 by Mr. G. R. Kraus, 33 by Professor M. S. Knebelman, and 44 by Dr. J. F. Randolph. Papers whose abstract numbers are followed by the letter *t* were read by title.

1. *Self-adjoint differential operators of elliptic type. Resolvent kernels*, by Dr. J. W. Calkin. (Abstract 43-9-323.)

2. *A further generalization of Lambert series*, by Rev. W. C. Doyle. (Abstract 43-5-268-*t*.)

3. *On the definition of differential operators*, by Dr. Kurt Friedrichs. (Abstract 43-9-329.)

4. *On the representation of bounded analytic functions by sequences of polynomials*, by Professor O. J. Farrell. (Abstract 43–9–327.)

5. *Bounds for parameters in n-noded solutions of Sturm-Liouville equations*, by Mr. J. K. L. MacDonald. (Abstract 43-9-345.)

6. *The measure of transitive geodesics on certain three-dimensional manifolds*, by Miss Annita Tuller. (Abstract 43-7-284.)

7. *Normal extensions of quartic fields with the symmetric group*, by Dr. D. M. Dribin (National Research Fellow). (Abstract 43-9-326.)

8. *An analog of the von Staudt-Clausen theorem*, by Professor Leonard Carlitz. (Abstract 43-7-291.)

9. *Applications of generalized quaternions to regularity of ternary quadratic forms* (preliminary report), by Professor Gordon Pall. (Abstract 43-7-302.)

10. *Groups with abelian central quotient group*, by Professor Reinhold Baer. (Abstract 43-9-316.)

11. *An application of Schläfli's modular equation to a conjec-*

ture of Ramanujan, by Professor D. H. Lehmer. (Abstract 43-7-299.)

12. *p-algebras over a field generated by one indeterminate*, by Professor A. A. Albert. (Abstract 43-7-287-*t*.)

13. *On existence of linear functionals defined over linear spaces*, by Professor R. P. Agnew. (Abstract 43-9-314.)

14. *On certain equations in matrices whose elements belong to a division algebra*, by Professor M. H. Ingraham. (Abstract 43-9-335.)

15. *On continuous transformations of bounded variation in the plane*, by Professor Tibor Radó. (Abstract 43-9-356-*t*.)

16. *Orthogonal polynomials in two complex variables*, by Professor Dunham Jackson. (Abstract 43-5-276.)

17. *An extension of Schwarz's lemma*, by Professor L. V. Ahlfors. (Abstract 43-5-265.)

18. *Some properties of formal deducibility*, by Professor H. B. Curry. (Abstract 43-9-325.)

19. *The expansion theory of ordinary differential systems of the first order*, by Professor R. E. Langer. (Abstract 43-9-340.)

20. *A note on the Cesàro method of summation*, by Dr. J. H. Curtiss. (Abstract 43-7-278.)

21. *On a certain class of non-linear expansions of an arbitrary analytic function*, by Dr. G. S. Ketchum and Dr. P. W. Ketchum. (Abstract 43-9-338.)

22. *Some theorems related to number theory*, by Professor J. J. Gergen. (Abstract 43-5-274-*t*.)

23. *The calculus of variations applied to Nörlund's sum*, by Professor Tomlinson Fort. (Abstract 43-7-295.)

24. *Theory of non-linear singular differential systems*, by Professor W. J. Trjitzinsky. (Abstract 43-9-364.)

25. *Asymptotic relations for derivatives*, by Dr. R. P. Boas, Jr. (National Research Fellow). (Abstract 43-7-289.)

26. *Linear equations with an infinity of unknowns*, by Mr. L. L. Lowenstein. (Abstract 43-9-343.)

27. *Displacement of equilibrium point of Green's function for an annulus*, by Dr. A. J. Maria. (Abstract 43-9-346.)

28. *Existence theorem for the flow of an ideal incompressible fluid in two dimensions*, by Dr. A. C. Schaeffer. (Abstract 43-7-308.)

29. *The role of the mean curvature in the immersion theory of surfaces*, by Mr. H. W. Alexander. (Abstract 43-9-315.)

30. *On a canonical form of the system of differential equations defining a ruled surface*, by Dr. W. L. Hutchings. (Abstract 43-9-333.)

31. *A classification of rational plane cubic curves*, by Mr. G. R. Kraus and Professor J. H. Neelley. (Abstract 43-7-277.)

32. *Spin representation of inversions*, by Dr. A. H. Taub. (Abstract 43-9-360.)

33. *Parallel vector fields*, by Professor M. S. Knebelman and Professor Walther Mayer. (Abstract 43-9-339.)

34. *On the metric differential geometry of a general surface in a linear space of four dimensions*, by Mr. C. E. Springer. (Abstract 43-7-311.)

35. *Plane peanian continua with unique maps on the sphere and in the plane*, by Professor V. W. Adkisson. (Abstract 43-7-286.)

36. *Perfectly compact Hausdorff spaces in which a normal space may be embedded*, by Professor R. G. Lubben. (Abstract 43-9-344.)

37. *Degenerate cycles bound*, by Professor A. W. Tucker. (Abstract 43-9-365.)

38. *A note on topological fields*, by Dr. Nathan Jacobson. (Abstract 43-9-336.)

39. *On periodic transformations of complexes*, by Dr. Moses Richardson. (Abstract 43-7-306.)

40. *Bounded sets in topological linear spaces*, by Mr. J. V. Wehausen. (Abstract 43-7-312.)

41. *An extensive class of analytic functions*, by Dean G. D. Birkhoff. (Abstract 43-9-366.)

42. *Analyticity of equilibrium figures of rotation*, by Dr. Bernard Friedman. (Abstract 43-7-296.)

43. *Superposition on monotonic functions*, by Mrs. E. M. Torrance. (Abstract 43-9-363.)

44. *Gillespie linear measure for point sets*, by Dr. A. P. Morse and Dr. J. F. Randolph. (Abstract 43-9-350.)

45. *The solution of linear boundary value problems in physics by means of the Laplace transformation*. II, by Professor R. V. Churchill. (Abstract 43-7-292.)

46. *On the solution of the differential equation* $u_{x_1x_1}+u_{x_2x_2}=u_{x_3x_3}+u_{x_4x_4}$, by Professor Fritz John. (Abstract 43-9-337.)

47. *On dependent probabilities*, by Mr. Garrett Birkhoff. (Abstract 43-9-321.)

48. *On completely continuous linear transformations* (preliminary report), by Mr. Ingo Maddaus, Jr. (Abstract 43-7-300-*t*.)

49. *Certain inequalities for functions of exponential type belonging to L along the real axis*, by Dr. D. L. Netzorg. (Abstract 43-9-352.)

50. *Representation of states in quantum mechanics by entire functions* (preliminary report), by Dr. D. C. Lewis. (Abstract 43-9-342.)

51. *Sets of conjugate matrices*, by Professor E. T. Browne. (Abstract 43-9-322.)

52. *The distribution of sums of squares of rank differences for the case of eight individuals*, by Professor E. G. Olds. (Abstract 43-9-354.)

53. *Variance of a general matching problem*, by Dr. J. A. Greenwood. (Abstract 43-9-331.)

54. *A graphical representation for the simple group of order 504*, by Professor H. S. M. Coxeter. (Abstract 43-7-293.)

55. *Extensions of linear functionals, with applications to limits, integrals, measures, and densities*, by Professor R. P. Agnew and Dr. A. P. Morse. (Abstract 43-5-273-*t*.)

56. *Abelian fields and duality of abelian groups*, by Professor Reinhold Baer. (Abstract 43-7-288-*t*.)

57. *Groups with preassigned central and central quotient group*, by Professor Reinhold Baer. (Abstract 43-9-317-*t*.)

58. *Halley's methods for the solution of equations*, by Professor Harry Bateman. (Abstract 43-9-318-*t*.)

59. *Remarks on the problem of Plateau*, by Dr. E. F. Beckenbach. (Abstract 43-9-319-*t*.)

60. *On a classification of integral functions by means of certain invariant point properties. A supplement*, by Dr. M. T. Bird. (Abstract 43-5-266-*t*.)

61. *Additive functions on Boolean algebras*, by Mr. Garrett Birkhoff. (Abstract 43-9-320-*t*.)

62. *Normal coordinates for extremals transversal to a manifold*, by Professor S. S. Cairns. (Abstract 43-7-290-*t*.)

63. *Combinatory logic as a semigroup* (preliminary report), by Professor Alonzo Church. (Abstract 43-5-267-*t*.)

64. *On tensors relative to the extended point transformation*, by Professor H. V. Craig. (Abstract 43-9-324-*t*.)

65. *Conformal transformations and the subspaces of a Riemann space*, by Dr. Aaron Fialkow. (Abstract 43-9-328-*t*.)

66. *The Riemannian curvature of a hypersurface*, by Dr. Aaron Fialkow. (Abstract 43-7-294-*t*.)

67. *A multiplier rule in abstract spaces*, by Dr. H. H. Goldstine. (Abstract 43-9-330-*t*.)

68. *On primitive groups of linear substitutions*, by Mr. Philip Hall. (Abstract 43-5-275-*t*.)

69. *Fixed points under homeomorphisms of continua which are not connected im kleinen*, by Mr. O. H. Hamilton. (Abstract 43-7-281-*t*.)

70. *On functional equations in linear topological spaces*, by Dr. D. H. Hyers. (Abstract 43-9-334-*t*.)

71. *Initial motion of a forced dissipative dynamical system under a sudden change in the force regime*, by Mr. W. H. Ingram. (Abstract 43-5-264-*t*.)

72. *On effective sets of points in relation to integral functions*. by Mr. V. G. Iyer. (Abstract 43-7-282-*t*.)

73. *The geometry of conformal symmetry* (*Schwarzian reflexion*) by Professor Edward Kasner. (Abstract 43-7-297-*t*.)

74. *The two conformal invariants of fifth order*, by Professor Edward Kasner. (Abstract 43-7-298-*t*.)

75. *On the growth of analytic functions*, by Dr. Norman Levinson. (Abstract 43-5-269-*t*.)

76. *Invariant manifolds near an invariant point of unstable type*, by Dr. D. C. Lewis. (Abstract 43-9-341-*t*.)

77. *Subrings of direct sums*, by Professor N. H. McCoy. (Abstract 43-7-283-*t*.)

78. *A note on an extension of Bernstein's theorem*, by Professor W. H. McEwen. (Abstract 43-9-347-*t*.)

79. *General conformal differential geometry*, by Professor A. D. Michal. (Abstract 43-9-348-*t*.)

80. *General projective differential geometry*. by Professor A. D. Michal. (Abstract 43-9-349-*t*.)

81. *An extension of Markoff's comparison theorems for zeros*

of orthogonal polynomials, by Dr. D. L. Netzorg. (Abstract 43-9-351-*t*.)

82. *The mean number of solutions of* $\phi(x)=n$, by Dr. D. L. Netzorg. (Abstract 43-9-353-*t*.)

83. *The maximum of a superharmonic function*, by Mr. Philip Newman. (Abstract 43-7-301-*t*.)

84. *The quaternion congruence* $\bar{l}at\equiv b \pmod{g}$ *and the equation* $h(8n+1)=x^2+y^2+z^2$, by Professor Gordon Pall. (Abstract 43-7-303-*t*.)

85. *On symmetric determinants*, by Professor W. V. Parker. (Abstract 43-7-304-*t*.)

86. *The characteristic roots of a matrix*, by Professor W. V. Parker. (Abstract 43-7-305-*t*.)

87. *Existence theorems for solutions of differential equations of non-integral order*, by Dr. A. E. Pitcher and Dr. W. E. Sewell. (Abstract 43-9-355-*t*.)

88. *On certain points in the theory of algebraic differential equations*, by Professor J. F. Ritt. (Abstract 43-9-357-*t*.)

89. *A quasi-analytic function which satisfies a functional equation*, by Mr. L. B. Robinson. (Abstract 43-9-358-*t*.)

90. *The nth prime is greater than* $n \log n$, by Dr. J. B. Rosser. (Abstract 43-7-307-*t*.)

91. *On a classification of metrical relations in the foundations of geometry*, by Dr. A. R. Schweitzer. (Abstract 43-7-309-*t*.)

92. *A theory of congruence in the foundations of geometry*, by Dr. A. R. Schweitzer. (Abstract 43-7-310-*t*.)

93. *The derivative of a polynomial on various curves of the complex plane*, by Dr. W. E. Sewell. (Abstract 43-7-279-*t*.)

94. *Spin representations of groups of motion of Riemannian spaces of constant curvature*, by Dr. A. H. Taub. (Abstract 43-9-361-*t*.)

95. *Ternary trilinear forms in the field of complex numbers*, by Dr. R. M. Thrall and Dr. J. H. Chanler. (Abstract 43-9-362-*t*.)

96. *The successive iterates of the Stieltjes kernel expressed in terms of the elementary functions*, by Professor D. V. Widder. (Abstract 43-7-280-*t*.)

97. *Decompositions of compact metric spaces*, by Professor R. L. Wilder. (Abstract 43-5-270-*t*.)

98. *Neighborhoods in locally connected spaces*, by Professor R. L. Wilder. (Abstract 43-5-272-*t*.)

99. *Property* S_n, by Professor R. L. Wilder. (Abstract 43-5-271-*t*.)

100. *The groups of order* 64 (preliminary report), By Mr. Philip Hall and Mr. J. K. Senior. (Abstract 43-9-332-*t*.)

101. *The generalized law of Bode as a mathematical principle*, by Professor O. E. Glenn. (Abstract 43-9-367-*t*.)

T. R. HOLLCROFT,
Associate Secretary

SHORTER NOTICES

Subharmonic Functions. By T. Radó. Berlin, Springer, 1937. iv+56 pp.

In concluding his review of the *Theorie der konvexen Körper* by Bonnesen and Fenchel (this Bulletin, vol. 41 (1935), pp. 613–614) the reviewer expressed his hope that a monograph on convex functions as such "will appear soon in the Ergebnisse series, and that it will prove just as exciting as the monograph by Bonnesen and Fenchel." The present monograph represents a realization of this hope, at least in so far as the theory of subharmonic functions is concerned. It is the first number of the fifth volume of the eminently useful series, *Ergebnisse der Mathematik und ihrer Grenzgebiete*, published by the Zentralblatt der Mathematik. A detailed treatment of the theory of convex functions which are a special case of subharmonic functions is promised in a subsequent monograph by W. Fenchel.

According to the author's statement the purpose of the present monograph "is first to give a detailed account of those facts which seem to constitute the general theory of subharmonic functions, and second to present a selected group of facts which seem to be well adapted to illustrate the relationships between subharmonic functions and other theories." The following list of contents gives some, though a very inadequate, idea of the material contained in the book. Chapter 1. Definitions and preliminary discussion, pp. 1–7. Chapter 2. Integral means, pp. 7–12. Chapter 3. Criterions and constructions, pp. 12–22. Chapter 4. Examples, pp. 22–31. Chapter 5. Harmonic majorants, pp. 31–40. Chapter 6. Representation in terms of potentials, pp. 40–46. Chapter 7. Analogies between harmonic and subharmonic functions, pp. 46–53. References, pp. 54–56.

A unified exposition of the general theory of subharmonic functions was badly needed owing to the fact that various authors used various apparently different (at least in form) definitions of subharmonic functions. The author's contributions toward this goal will be welcomed by general readers and will be particularly valuable for specialists who have to use subharmonic functions as a tool of research. The applications treated in the book are numerous and the topics are wisely selected. The reviewer feels however that the number of pages devoted to this purpose might have been gainfully increased. The book contains a considerable amount of new material (both new results and new and better proofs of older results) which never has been published before and which is due to several mathematicians (mainly to Evans, Saks, and the author himself).

The exposition is elegant and clear throughout the book, although of necessity condensed in some places, the amount of hints and leading remarks being always sufficient, however, to allow an attentive reader to reconstruct a complete proof. The only exception to the above which came to the reviewer's attention is the proof of the theorem in 2.16 (p. 10). The proof of this theorem is based on a statement to the effect that a sufficient condition for $\log f(\rho)$ to be convex in $\log \rho$, $\rho_1<\rho<\rho_2$, is that $\rho^\alpha f(\rho)$ be convex in $\log \rho$ for each $\alpha>0$,

with reference to a paper by F. Riesz. In this form the statement is not correct, as can be shown by simple examples such as $f(\rho)=\log \rho$. The statement becomes correct if α is assumed to be any real number, and this assumption is explicitly used in F. Riesz' argument. Incidentally there is an obvious misprint in the page reference here and at the end of the book, there being no paper of F. Riesz on the allotted pages 3–8! On the whole the book contains remarkably few misprints, and the list of references is very inclusive. In fact the only omission that the reviewer was able to discover is an extensive memoir by N. Günther, *Sur les intégrales de Stieltjes et leur applications aux problèmes de la physique mathématique*, Travaux de l'Institut Physico-Mathématique Stekloff, vol. 1 (1932), Ch. 8, which contains, among many other things, a discussion of F. Riesz' theorem concerning the representation of superharmonic functions in terms of potentials of positive mass distributions and of harmonic majorants.

The author and the Ergebnisse series should be congratulated upon publishing this exceedingly interesting and valuable monograph, which will prove indispensable for anyone who works in the field of the theory and applications of subharmonic functions.

J. D. TAMARKIN

Einführung in die analytische Geometrie und Algebra. Volume 2. By O. Schreier and V. Sperner. Leipzig and Berlin, Teubner, 1935. 308 pp.

The first volume of this work, bearing the same title, was published in 1931 and reviewed in this Bulletin (vol. 38 (1932), p. 622). The main feature (complete fusion of the foundations of geometry and algebra) and the excellent qualities of the book, were sufficiently pointed out in the review of the first volume, so that at present we restrict ourselves merely to giving a short list of contents. Chapter 1 (Elements of the theory of groups) includes Abelian groups. Chapter 2 (Linear transformations, matrices) deals primarily with the theory of elementary divisors, orthogonal transformations, symmetric and Hermitian matrices and the like. Chapter 3 (Projective geometry) contains an excellent exposition of the foundations of projective geometry in n-dimensional spaces. After a perusal of this second volume the reviewer feels confirmed in his previous opinion that the book could be successfully used as a text or reference book in a graduate course in this country.

J. D. TAMARKIN

NOTES

In addition to the members of the editorial staff of the Bulletin, the following persons have assisted the editors, either by serving as referees or by advising concerning papers offered for publication in the present volume: R. G. Archibald, M. A. Basoco, E. T. Bell, B. A. Bernstein, Garrett Birkhoff, Salomon Bochner, Leonard Carlitz, J. A. Clarkson, Richard Courant, A. T. Craig, H. B. Curry, J. L. Doob, Arnold Dresden, Arnold Emch, H. T. Engstrom, G. C. Evans, Orrin Frink, T. C. Fry, H. L. Garabedian, W. C. Graustein, Einar Hille, T. R. Hollcroft, Harold Hotelling, E. V. Huntington, W. A. Hurwitz, M. H. Ingraham, R. D. James, E. R. van Kampen, R. E. Langer, Lincoln La Paz, Solomon Lefschetz, D. H. Lehmer, E. J. McShane, C. C. MacDuffee, W. A. Manning, H. H. Mitchell, C. N. Moore, H. M. Morse, F. J. Murray, John von Neumann, Oystein Ore, H. A. Rademacher, G. Y. Rainich, W. T. Reid, J. F. Ritt, J. A. Shohat, E. B. Stouffer, J. D. Tamarkin, J. M. Thomas, T. Y. Thomas, E. W. Titt, W. J. Trjitzinsky, R. J. Walker, H. S. Wall, J. L. Walsh, Morgan Ward, J. H. M. Wedderburn, Louis Weisner, W. M. Whyburn, Norbert Wiener, B. C. Wong, Leo Zippin, Max Zorn. The editors desire publicly to acknowledge this service.

The School of Mathematics of the Institute for Advanced Study each year allocates a small number of stipends to gifted young mathematicians and mathematical physicists to enable them to study and to do research work at Princeton. Candidates must have given evidence of ability in research comparable at least with that expected for the degree of Doctor of Philosophy. Blanks for application may be obtained from the School of Mathematics of the Institute, Fine Hall, Princeton, N. J., and are returnable by February 1, 1938.

Professor J. W. Alexander, of the Institute for Advanced Study, Professor D. J. Struik, of the Massachusetts Institute of Technology, and Professor Louis Weisner, of Hunter College, have been added to the staff of contributing editors of Science and Society.

The Mittag-Leffler medal of the Mathematical Institute at Stockholm has been awarded to Dr. David Hilbert, professor of mathematics at the University of Göttingen.

Professor Solomon Lefschetz, of Princeton University, has been elected a corresponding member of the Reale Accademia di Scienze, Lettere ed Arti di Padova.

Associate Professor W. E. Byrne, of Virginia Military Institute, has been promoted to a professorship.

Assistant Professor Leonard Carlitz, of Duke University, has been promoted to an associate professorship.

Professor W. B. Carver, of Cornell University, is on leave of absence for the first semester and is traveling in Europe.

Professor T. F. Cope, of Marietta College, has been appointed to an assistant professorship at Queens College, Flushing, N. Y.

Assistant Professor A. H. Copeland, of the University of Michigan, has been promoted to an associate professorship.

Associate Professor C. C. Craig, of the University of Michigan, is on leave of absence for the year 1937–38, studying at the Galton Laboratory, University College, London, England.

Professor Eleanor C. Doak, of Mount Holyoke College, has retired.

Dr. P. S. Dwyer has been appointed to an assistant professorship at the University of Michigan.

Dr. R. L. Echols has been appointed to an assistant professorship in physics at the United States Naval Post-graduate School, Annapolis.

Assistant Professor L. R. Ford, of the Rice Institute, has been appointed professor of mathematics and chairman of the department of mathematics at the Armour Institute of Technology.

Dr. Kurt Friedrichs has been appointed visiting professor of applied mathematics at New York University.

Associate Professor J. J. Gergen, of Duke University, has been appointed chairman of the department of mathematics.

Mr. O. H. Hamilton, of the University of Texas, has been appointed to an assistant professorship at Oklahoma Agricultural and Mechanical College.

Rev. B. A. Hausmann, S.J., has been appointed to a professorship at the University of Detroit.

Dr. E. A. Hedberg has been appointed professor of mathematics and head of the department of mathematics at the University of South Dakota.

Dr. Ralph Hull has been appointed to an assistant professorship at the University of Illinois.

Dr. H. S. Kaltenborn has been appointed to an assistant professorship at the Philadelphia College of Pharmacy and Science.

Dr. E. C. Kennedy has been appointed to an associate professorship at the Texas College of Arts and Industries.

Professor W. J. Kirkham has been granted a year's leave of absence from Oregon State College to do statistical research in Washington, D. C.

Professor H. I. Lane, of the University of South Dakota, has been appointed to a professorship at Hendrix College, Conway, Ark.

Canon Georges Lemaître, professor of mathematics and theoretical physics at the University of Louvain, Dr. Kurt Gödel, of the University of Vienna, and Professor Emil Artin, of the University of Hamburg, have joined the faculty of the University of Notre Dame.

Assistant Professor Marie Litzinger, of Mount Holyoke College, has been promoted to an associate professorship and appointed chairman of the department of mathematics.

Dr. L. L. Lowenstein has been appointed to an assistant professorship at Alfred University.

Dr. Roy MacKay has been appointed to an associate professorship at the Junior College, Portales, N. M.

Dr. L. E. Mehlenbacher has been appointed professor of mathematics and head of the department of mathematics at Arizona State Teachers College.

Associate Professor W. O. Menge, of the University of Michigan, has resigned to become associate actuary of The Lincoln National Life Insurance Company, Fort Wayne, Indiana.

Dr. A. C. Olshen has been appointed actuary in the Oregon State Insurance Commission.

Assistant Professor H. H. Pixley, of Wayne University, has been promoted to an associate professorship.

Dr. D. P. Richardson, of the University of Arkansas, has been promoted to an associate professorship.

Assistant Professor J. H. Roberts, of Duke University, has been granted a leave of absence for the year 1937–38. During this time he will be a visiting lecturer at Princeton University.

Assistant Professor Rafael Sanchez-Diaz, of the University of Puerto Rico, has been promoted to an associate professorship.

Associate Professor J. B. Scarborough, of the United States Naval Academy, Annapolis, has been promoted to a professorship.

Dr. Wladimir Seidel, of the University of Rochester, has been promoted to an assistant professorship.

Dr. L. W. Sheridan has been appointed to a professorship at the College of Mount Saint Vincent.

Mr. C. E. Smith has been appointed to an assistant professorship in astronomy at the San Diego State College, San Diego, Calif.

Assistant Professor J. J. Stoker, of the Carnegie Institute of Technology, has been appointed to an assistant professorship at New York University.

Associate Professor M. H. Stone, of Harvard University, has been promoted to a professorship.

Dr. E. C. Stopher, of the University of Iowa, has been appointed assistant professor and head of the department of mathematics at Ashland College, Ashland, Ohio.

Dr. Gerhard Tintner, of the Cowles Commission, has been appointed assistant professor of mathematics and economics at Iowa State College.

Dr. E. W. Titt has been appointed to an assistant professorship at the University of Maryland.

Professor J. H. Van Vleck, of Harvard University, has been appointed visiting professor at Princeton University for the first half of the present academic year.

Associate Professor D. V. Widder, of Harvard University, has been promoted to a professorship.

Associate Professor F. E. Wood, of Northwestern University, is on leave of absence for the academic year 1937–38.

The following members of the American Mathematical Society are in residence at the Institute for Advanced Study this academic year: Dr. Miriam F. Becker; Dr. P. G. Bergmann; Professor Marie M. Johnson, of Oberlin College Dr. P. W. Ketchum; Professor Harry Levy, of the University of Illinois; Professor N. H. McCoy, of Smith College; Professor C. C. MacDuffee, of the University of Wisconsin; Professor C. B. Morrey, of the University of California; Dr. A. P. Morse; Dr. C. J. Nesbitt; Professor Rufus Oldenburger, of the Armour Institute of Technology; Dr. J. F. Randolph; Dr. Moses Richardson; Dr. Hyman Serbin; Dr. M. F. Smiley; Dr. Frank Smithies; Dr. C. B. Tompkins; Professor H. S. Wall, of Northwestern University; Professor Aurel Wintner, of the Johns Hopkins University.

The following mathematicians from foreign universities are also in residence at the Institute: Dr. Valentin Bargmann; Dr. Achille Bassi, of the University of Florence; Professor Erich Hecke, of the University of Hamburg (second term); Professor Vaclav Hlavatý, of the University of Prague; Dr. Witold Hurewicz, of the University of Amsterdam; Dr. Leopold Infeld, of the University of Lwow; Professor Walther Mayer, of the University of Vienna; Fr. Tadasi Nakayama, of the University of Tokyo; Professor M. H. A. Newman, of St. John's College, Cambridge, England; Dr. W. J. van Stockum.

The following appointments to instructorships are announced: Agricultural and Mechanical College of Texas: Dr. H. A. Luther; University of Alabama: Dr. H. C. Ayres; Brown University: Mr. J. V. Wehausen; California Institute of Technology: Dr. E. W. Paxson; University of California: Dr. R. M. Robinson; University of California at Los Angeles: Mr. F. A. Valentine; University of California Extension Division: Dr. P. M. Singer; Campbell College: Mr. R. E. Smith; Carleton College: Dr. C. Grace Shover; College of the City of New York: Dr. G. G. Harvey; University of Delaware: Dr. G. C. Webber; Drexel Institute: Mr. W. B. Campbell; Florida Southern College: Mr. A. R. Turquette; University of Illinois: Dr. C. W. Mendel, Dr. H. E. Vaughan; Indiana Uni-

versity: Mr. Louis Green; Iowa State College: Mr. R. H. Cook, Mr. H. C. Fryer; Marietta College: Dr. Theodore Bennett; University of Michigan: Dr. T. N. E. Greville, Dr. M. L. Kales, Mr. E. D. Rainville, Dr. R. M. Thrall; State University of Montana: Dr. Harold Chatland; Mount Holyoke College: Miss Dorothy L. Bernstein; University of Notre Dame: Dr. A. N. Milgram; Ohio State University: Dr. C. E. Rhodes, Dr. J. W. T. Suckau; Oregon State Agricultural College: Dr. O. G. Harrold; Queens College, Flushing, N. Y.: Dr. Banesh Hoffman, Dr. Lulu Hofmann, Dr. H. W. Raudenbush, Dr. Arthur Sard; Rensselaer Polytechnic Institute: Mr. R. E. Street; Rice Institute: Dr. Walter Leighton; South Dakota School of Mines: Mr. A. W. Davis; University of Tennessee: Mr. M. J. Turner; University of Utah: Miss Harriet Rees, Dr. Anna A. Stafford; University of Wisconsin: Mr. Churchill Eisenhart, Dr. Bernard Friedman, Dr. D. H. Hyers, Dr. S. B. Jackson, Mr. R. B. Kershner, Mr. R. W. Wagner.

Dr. E. B. Elliott, emeritus professor of pure mathematics in the University of Oxford, and Fellow of the Royal Society, died on July 21, 1937, at the age of eighty-six years.

Dr. R. P. Baker, associate professor of mathematics at the University of Iowa, and a member of the Convocation of the University of London, died on August 13, 1937, at the age of seventy-one years. He had been a member of the Society since 1904.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

372. Professor R. P. Agnew: *Linear functionals satisfying prescribed conditions.*

Let P be the class of p-functions $p=p(x)$ and F the class of linear functionals $f=f(x)$, defined over a linear space E. A functional $p\varepsilon P$ is said to *enforce* a set S of properties if each $f\varepsilon F$ with $f(x)\leqq p(x)$ for all $x\varepsilon E$ has the properties S. It is shown that $p\varepsilon P$ exists which enforces S if and only if $f\varepsilon F$ exists having properties S, and that $p_2\varepsilon P$ enforces all the properties which $p_1\varepsilon P$ enforces if and only if $p_2(x)\leqq p_1(x)$ for all $x\varepsilon E$. The main problem of this paper is that of characterizing analytically the class of $p\varepsilon P$ which enforces a preassigned property or set of properties. The problem is solved for the property 1^0: $f(x)\leqq p_0(x)$, $p_0\varepsilon P$ being preassigned; for the property 2^0: $f(y)=f(x)$ for all pairs $\{x, y\}$ belonging to a preassigned set Ψ of pairs; and for the property 3^0: both 1^0 and 2^0. The analytic criteria are discussed for the case where Ψ is the set of all pairs $\{x, g(x)\}$ with $x\varepsilon E$, and $g\varepsilon G$ where G is a group of linear transformations mapping E into itself, and for the further special case where G is solvable. Applications to generalized limits and integrals are given. (Received September 25, 1937.)

373. Dr. R. P. Boas and Professor D. V. Widder: *The iterated Stieltjes transform.*

The iterated Stieltjes transform is (1) $f(x)=\int_{0+}(x+u)^{-1}du\int_{0+}^{\infty}(u+t)^{-1}d\alpha(t)$. It is formally equivalent to the S_2-transform (2) $f(x)=\int_{0+}^{\infty}\log(x/t)(x-t)^{-1}d\alpha(t)$; this formal equivalence is not always valid, but (2) can always be written in the form (1). The transform (1) is inverted by the use of a differential operator $H_{k,t}[f(x)]=L_{k,t}L_{k,t}[f(x)]$, where $L_{k,t}[f(x)]$ is the inversion operator for the Stieltjes transform, introduced by D. V. Widder (abstract 43-1-87). If $\alpha(t)$ is the integral of a function $\phi(t)$, $H_{k,t}[f(x)]$ approaches $\phi(t)$ for almost all positive t, $(k\to\infty)$. If $\alpha(t)$ is a normalized function, of bounded variation on every interval (ϵ, R), $(0<\epsilon<R<\infty)$, the integral of $H_{k,t}[f(x)]$ on $(0, t)$ approaches $\alpha(t)-\alpha(0+)$ for every positive t. Necessary and sufficient conditions for a given $f(x)$ to have the representations (1) and (2) with $\alpha(t)$ belonging to various functional classes are obtained in terms of $H_{k,t}[f(x)]$. For a preliminary report on the S_2-transform, see D. V. Widder, Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 242–244. (Received September 18, 1937.)

374. Professor Richard Brauer and Dr. C. J. Nesbitt: *On modular representations of groups of finite order.*

Any irreducible representation of a group G of finite order g is equivalent to a representation A whose coefficients are integers of an appropriate algebraic number field K. If p is a rational prime, we obtain a modular representation $A^{(p)}$ of G on considering A modulo a prime ideal of K which divides p. It is well known that $A^{(p)}$ is irreducible if p does not divide the group order g. This result will be extended in the following way to the singular case where p divides g. Let p^a be the highest power of p dividing g. If p^a divides the degree of A, then $A^{(p)}$ is an irreducible modular representation. Two non-equivalent ordinary representations A_1 and A_2 which satisfy the assumption yield non-equivalent modular representations. No such $A^{(p)}$ appears as irreducible constituent of an indecomposable modular representation U (except for $U=A^{(p)}$). (Received September 25, 1937.)

375. Professor R. H. Cameron: *The distribution of values of an analytic almost periodic function in equally spaced circles.*

This paper deals with functions analytic in a strip and almost periodic in a substrip. A system of equally spaced circles is drawn with their centers on the edge of the almost periodic strip, and it is shown that the function takes on values arbitrarily close to a given value A in a relatively dense set of these circles. If the distance between the circles belongs to the module of the function, an extra restriction on the set of circles is necessary. (Received October 4, 1937.)

376. Dr. J. H. Curtiss: *Uniform convergence and summability of the Jacobi series on an unrestricted lemniscate.*

Integrals of the Dirichlet and Fejér type are set up for the real and imaginary parts of the Jacobi interpolation series, and are studied in some detail. It is found, for example, that if the boundary values $F(z)$ of the sum function are continuous on the lemniscate of convergence, then the Jacobi series is summable (C, α), $\alpha>0$, to $F(z)$ uniformly on the lemniscate of convergence, regardless of whether or not the lemniscate has multiple points. Uniform convergence occurs if the boundary values are continuous and of bounded variation. The degree of uniform convergence corresponding to a Lipschitz condition is also investigated by real variable methods, and examples are given to show that the results are the best possible. (Received October 4, 1937.)

377. Mr. J. J. DeCicco: *Fields whose geodesic series have circles for their point- or line-unions.*

This paper is a continuation of the two papers, *The group of turns and slides and the geometry of turbines*, by Kasner, American Journal of Mathematics, vol. 33 (1911), *The geometry of turbines, flat fields, and differential equations*, by Kasner and the author, American Journal of Mathematics, vol. 59 (1937), and abstract 43-5-260, *The differential geometry of series of lineal elements*, by the author. A series of a field is called a *geodesic series* when its curvature at any

element is less than the curvature of the other field series which pass through that element and have the same tangent turbine. The main results are: 1. For ∞^2 circles to be the point curves of the geodesic series of a field, it is necessary and sufficient that they all pass through a fixed point. Moreover the field is a non-linear flat field. 2. For ∞^2 circles to be the line curves of the geodesic series of a field, it is necessary and sufficient that they be tangent to a fixed (linear or non-linear) circle. (Received September 27, 1937.)

378. Professor Philip Franklin: *Note on the four color problem.*

Recently, the author announced the theorem that every regular map, irreducible with respect to four colors, contains at least 32 regions. In view of certain new reducible configurations and results due to C. E. Winn, the proof of this theorem is considerably simplified. (Received September 24, 1937.)

379. Dr. V. V. Latshaw: *On fourth order self-adjoint difference systems.*

This note includes three theorems: one on adjoint functions, one on self-adjoint boundary conditions, and one on characteristic values. (Received September 30, 1937.)

380. Dr. A. N. Lowan: *Note on the operational determination of Green's functions in the theory of heat conduction.*

This note is a sequel to a paper appearing in the July issue of the Philosophical Magazine and bearing a similar title. It deals with the determination of the Green's function for a cylinder and for the space bounded internally by a cylinder, the boundary condition in both cases being in the form $(\partial/\partial r+h)u=0$ from which the corresponding solutions for the cases where the boundary is either impervious to heat or kept at 0 degrees is obtained by putting $h=0$ and $h=\infty$, respectively. The Green's function is constructed by the superposition of the solution of the differential equation of heat conduction satisfying the condition for a line source and one vanishing at $t=0$. The Laplace transforms of the last two solutions are first obtained, the arbitrary constants being determined so as to satisfy the boundary condition. By means of Neumann's expansion and some known identities involving integrals of Bessel's functions, the sum of the last two Laplace transforms is written in the form of an integral whose inverse Laplace transform is readily obtained. (Received September 29, 1937.)

381. Professor Deane Montgomery and Dr. Leo Zippin: *On abelian groups of auto-homeomorphisms of metric spaces.*

This note is concerned with compact abelian groups of auto-homeomorphisms of a metric space E from the point of view, particularly, of the "orbit-space." It gives a useful characterization of the dimension of compact abelian groups and it contains the following quite special theorem: the topological rotation group of the euclidean 3-space E is characterized as an abelian compact connected group of auto-homeomorphisms of E. (Received October 2, 1937.)

382. Professor Tadasi Nakayama: *A remark on representations of groups.*

In this paper the Frobenius theorem on induced characters of finite groups is extended to the case of bounded representations of general groups. The extension is based on the fact that, with a suitable definition of compositions, we can consider the ring of almost periodic functions on a subgroup (of a not necessarily finite index) of a group as a left or right operator-ring of that of the whole group. (Received September 29, 1937.)

383. Professor Rufus Oldenburger: *Decomposition of forms.*

Sylvester, Dickson, Bronowski, and others studied the representation of forms by sums of powers of linear forms. In the present paper it is proved that, for a field with p or more elements, every form F of degree p with symmetric matrix can be written as a sum $S=aL^p+\cdots+dM^p$, where there are a finite number N of forms in the set $L, \cdots, M$ of linear forms, and $a, \cdots, d$, are in the given field. For a field with less than p elements such a representation of F is not, in general, possible. In another paper, not yet published, it was proved that a form F, for a field for which the symmetric matrix of F is unique, can be represented by a sum S where $L, \cdots, M$ are linearly independent if and only if a determinant rank of F and a new type of invariant called "factorization rank," of F are equal. For a given form F and value of N the representation S is not unique. The relations between representations of a given form F by sums of type S, where $L, \cdots, M$ are linearly independent, can be expressed in terms of diagonal and permutation matrices. An analogous theory holds for multilinear forms. (Received October 4, 1937.)

384. Dr. B. J. Pettis: *A note on regular Banach spaces.*

Several conditions can be listed as necessary and sufficient that a Banach space $\mathfrak{X}$ be regular, that is, that given X in $\overline{\mathfrak{X}}$ there is an x in $\mathfrak{X}$ such that $X(f)=f(x)$ for all f in $\overline{\mathfrak{X}}$: for example, regularity of $\overline{\mathfrak{X}}$ is equivalent to that of $\mathfrak{X}$; separable $\mathfrak{X}$ is regular if and only if $\mathfrak{X}$ is weakly complete and either $\overline{\mathfrak{X}}$ is separable or $\mathfrak{X}$ is weakly compact. As a corollary, in the space of bounded additive functions (of L-measurable sets) satisfying condition (N) of Lusin the completely additive (N) functions form a closed set of the first category. The space of completely continuous operations from regular $\mathfrak{X}$ to l^1 (absolutely convergent series) is equivalent to that of the unconditionally convergent series in $\overline{\mathfrak{X}}$; if in addition $\mathfrak{X}$ is separable this also gives the space of linear operations from $\mathfrak{X}$ to l^1. The functions of bounded variation from a linear interval to a given $\mathfrak{X}$ are all strongly differentiable almost everywhere if and only if all are weakly differentiable almost everywhere; hence if $\mathfrak{X}$ is separable and regular, every BV function from the line to $\mathfrak{X}$ is strongly differentiable almost everywhere and its derivative is Bochner-integrable. (Received September 7, 1937.)

385. Mr. L. B. Robinson: *On a functional equation which admits a lacunary function as solution.*

The author has studied the equation $u'(x)=\lambda/(a-x^2)u(x^2)$ when $a=1$. When $a\neq 1$, the solution converges within and without the unit circle. It also

admits the circumference of the unit circle as a natural boundary. Moreover each point of the circumference is a point of accumulation in whose neighborhood can be found an infinity of singular points. (Received August 26, 1937.)

386. Dr. J. F. Randolph: *The Vitali covering theorem for Carathéodory linear measure.*

Results on the same problem were presented to the Society under a similar title in January, 1936 (abstract 42-1-45). In this paper, the same results are obtained under weaker conditions, and necessity as well as sufficiency is discussed. (Received October 5, 1937.)

387. Dr. A. R. Schweitzer: *Remarks on the foundations of geometry.*

The author's remarks concern first the existence of additions to his sets of generating relations for descriptive and metrical geometry, such as angular analogs of the author's relations S, B, I, expressing respectively angular "incidence" and "non-incidence" and "point in the interior of the angle α_{jn}" (see abstract 43-1-92), and the corresponding metrical relations expressing angular bisection. The remaining part of the paper is devoted to the n-dimensional generalization of the archimedean postulate in terms of circles, spheres, and so on, and the corresponding statement of (so-called pure) continuity. A critical analysis is made of the postulates of non-archimedean continuity of Veronese, Hilbert, and Vahlen. In particular, Hilbert's "axiom of completeness" is replaced by an analogous but possibly more concise postulate which emphasizes the non-adjoinability of existential axioms rather than the non-addition of geometric elements. From the point of view of this replacement, Hilbert's concept of completeness has interesting anticipatory analogies in the literature of antiquity. (Received October 18, 1937.)

388. Professor Tibor Radó: *A theorem on generalized Jacobians.*

Let the continuous transformation T be defined by the equations $x=x(u, v)$, $y=y(u, v)$, $0\leq u\leq 1$, $0\leq v\leq 1$. Suppose T is absolutely continuous (as defined by Banach, Fundamenta Mathematicae, vol. 7 (1925), pp. 225–236), and denote by $J(u, v)$ the generalized Jacobian introduced by Schauder (the product $i(u, v)\ D(u, v)$ on p. 71 of his paper in Fundamenta Mathematicae, vol. 12 (1928), pp. 47–74). Suppose finally that the partial derivatives x_u, x_v, y_u, y_v exist almost everywhere. The purpose of this paper is to prove that one has then $J(u, v)=x_uy_v-x_vy_u$ almost everywhere. (Received October 7, 1937.)

389. Professor Akitsugu Kawaguchi: *Geometry in an n-dimensional space with the arc length* $s=\int(A_{ij}x'^{i}x'^{j}+B)^{1/p}dt$.

In this paper a theory is developed in a special Kawaguchi space in which the arc length is given by $s=\int(A_{ij}x'^{i}x'^{j}+B)^{1/p}dt$. At first the intrinsic connections C_1 and C_2, which are invariant under any parameter transformation $\bar{t}=\bar{t}(t)$, are considered and, by use of these connections, the covariant differentiations,

curvature, and torsion tensors are then introduced. The geodesic and minimal curves are also discussed and their geometrical interpretations are added. Finally many other important and fundamental formulas, such as generalized Bianchi's identities, are also completely calculated. (Received October 6, 1937.)

390. Mr. P. R. Halmos: *Note on almost-universal forms.*

Ramanujan and Dickson proved that there are 54 universal forms $ax^2+by^2+cz^2+dt^2$, with positive integral coefficients a, b, c, d. It is the purpose of this note (which is to appear shortly in this Bulletin) to investigate *almost—universal forms*, that is, to exhibit sets of positive integral coefficients a, b, c, d such that $ax^2+by^2+cz^2+dt^2$ represents every positive integer with exactly one exception. A set of 88 forms is exhibited which has to contain all almost-universal forms. By a method involving the regularity of ternary forms it is shown that 86 of these forms are almost-universal. The method breaks down in the case of the forms $x^2+2y^2+7z^2+11t^2$ and $x^2+2y^2+7z^2+13t^2$ because of the essential irregularity of all the ternary forms involved. (Received October 14, 1937.)

391. Professor C. H. Forsyth: *Extension formulas for interpolation with leading differences.*

Interpolation by leading differences is very valuable in interpolating whole series of values, not only because of the labor saved thereby, but also because of the valuable running check upon the work provided from interval to interval. However, such work calls for the separate computation of these leading differences for each interval. The author develops extension formulas for continuing the work from interval to interval with much less labor since these extension formulas are of a higher order of differences and therefore much simpler. Formulas are developed to cover both ordinary interpolation and interpolation with groups of given values. (Received October 19, 1937.)

392. Dr. E. F. Beckenbach: *A relative of the lemma of Schwarz.*

It is shown that if $p(r, \theta)$ is non-negative, and $\log p(r, \theta)$ is subharmonic, for $r<1$, then $q(r, \theta)\equiv\int_0^r p(\rho, \theta)d\rho$ also has a subharmonic logarithm for $r<1$. It follows that if $w=f(z)$ is analytic for $|z|<1$, and if for every θ, $l(1, \theta)\leqq 1$, where $l(r, \theta)$ is the length of the image on the w-plane of the radius from $(0, 0)$ to (r, θ) on the z-plane, then $l(r, \theta)\leqq r$. A generalization to conformal maps on surfaces of negative Gaussian curvature is obtained. (Received October 12, 1937.)

393. Dr. W. C. Randels: *On the weak summability of Fourier series.*

A method of summability defined by a matrix $\{a_{mn}\}$ is said to be weakly effective if the sequence of transforms $T_m(f, x)=\sum a_{mn}f_n e^{inx}$ of the Fourier series of $f(x)$ converges weakly in L_p to $f(x)$. It is shown that weak effectiveness is equivalent to the strong effectiveness defined by Hille and Tamarkin. (Received October 12, 1937.)

394. Professor R. V. Churchill: *Additional notes on the inversion of the Laplace transformation.*

In a recent paper (Mathematische Zeitschrift, vol. 42 (1937), pp. 567–579) the author gave criteria, useful in the solution of boundary value problems, for the representation of the inverse Laplace transform by a series. These are conditions for the validity of an expansion formula which is a generalized form of one of Heaviside's formulas. Conditions which are more useful in problems involving differential equations of parabolic type are established and illustrated in the present note. (Received October 12, 1937.)

395. Professor A. A. Albert: *A quadratic form problem in the calculus of variations.*

The paper gives the first proof of a conjecture of Professor G. A. Bliss which arose in connection with a sufficiency proof in the calculus of variations. Let f and g be real quadratic forms in n real variables such that f is positive for all values of the variables making $g=0$. Then there exists a real r such that $f+rg$ is positive definite. The range for r is also determined in terms of the characteristic roots of the matrix of the pencil $f+rg$. (Received October 13, 1937.)

396. Dr. H. H. Goldstine: *Weakly complete Banach spaces.*

The author extends the definition of weakly complete spaces by means of the Moore-Smith limit and investigates the problem of determining conditions for a Banach space to be weakly complete. A necessary and sufficient condition is that the space be equivalent in the sense of Banach to the adjoint of its adjoint. Moreover, when the adjoint of the original space is separable the definition of weak completeness given by the author reduces to the one ordinarily stated. (Received October 16, 1937.)

397. Professor L. M. Blumenthal: *The characterization of pseudo-$S_{2,r}$-sets.*

In this paper is solved the problem proposed by Karl Menger of characterizing those pseudo-$S_{2,r}$-sets containing more than five points, no pair of points being diametral (that is, with a distance equal to πr). It is proved that the distance of every pair of distinct points of such a set equals $r\cos^{-1}(\pm 1/3)$, with the negative sign holding for a number $\mu>0$ of the distances. As a corollary of this result the following determinant theorem is proved: If the determinant $\Delta=|r_{ij}|$, $r_{ij}=r_{ji}$, $-1<r_{ij}<1$, $(i\neq j)$, $r_{ii}=1$, $(i, j=1, 2, \cdots, m; m>5)$, is such that (i) every third-order principal minor is non-negative, (ii) every principal minor of order four vanishes, (iii) at least one fifth-order principal minor is not zero, then (1) every third-order principal minor is positive, (2) every principal minor of order five has the value $-(4/3)^4/3$, (3) the absolute value of each element of Δ outside the principal diagonal is $1/3$, (4) upon multiplying rows and the same numbered columns of Δ by -1, each element of Δ outside the principal diagonal takes the value $-1/3$, (5) $\Delta=-(4/3)^{m-1}(m-4)/3$. (Received October 27, 1937.)

398. Dr. G. M. Ewing: *Some implicit function theorems in the large.*

Sufficient conditions for $f(x, y)=0$ to define y as a function of x in the neighborhood of a particular point have been given in many forms. In the present paper theorems are obtained which insure the existence of the implicit function over a specified domain (x_1, x_2) of the independent variable. The method used involves a suitable adaptation of the Hilbert construction used in existence theorems in the calculus of variations. The paper also includes a generalization to the case of n implicit functions determined by n equations. (Received October 27, 1937.)

399. Dr. P. O. Bell: *On a family of curves associated with a conjugate net.*

The author introduces a family of curves which corresponds to an arbitrarily selected conjugate net on a non-ruled analytic surface. This family is studied in detail, and particularly with reference to its relation to other families. A number of interesting results are obtained, of which the following is typical: A geometric characterization is obtained for a two-parameter family of curves whose cusp-axis is the axis of Cech. Simple geometric specializations present the curves of Segre and the curves of Darboux as unique curves of this family. (Received October 27, 1937.)

400. Professor Dunham Jackson: *Note on orthogonal polynomials in three variables.*

This is a supplement to an earlier paper by the author on the same subject (abstract 43-1-31), relating particularly to the convergence of developments in series of the orthogonal polynomials, and a proof of the uniqueness of the weight function for a given orthogonal system. (Received October 28, 1937.)

401. Mr. J. H. Daoust: *On certain constants associated with Bernstein's theorem.*

An extension of Bernstein's theorem on the derivative of a polynomial or a trigonometric sum, relating to the behavior of a trigonometric sum in an interval (a, b) of length less than a period, has been stated (D. Jackson, Transactions of this Society, vol. 40 (1936), p. 227) in a form involving a constant C which depends only on the length of the interval. The present paper shows that this theorem is true with a value of C which is not dependent even on a and b, but is an absolute constant. Some other theorems of similar character are rendered more precise in a corresponding manner. (Received October 28, 1937.)

402. Mr. Fulton Koehler: *Orthogonal polynomials on a square.*

This paper is a study of formal properties of polynomials orthogonal on the perimeter of a square as a special case of polynomials in two real variables orthogonal on an algebraic curve. The polynomials of the orthogonal system with unit weight function are found to be expressible explicitly in terms of Jacobi polynomials. The application of the formal results to problems of convergence will be dealt with in a later paper. (Received October 28, 1937.)

403. Dr. R. B. Kershner: *On a van der Corput absolute constant.*

According to van der Corput, there exist absolute constants μ, ν such that if $f(x)$ is a real function for which $f''(x) \geqq r > 0$ in $[a, b]$, then (1) $\left|\int_a^b \cos f(x)dx\right| \leqq \mu/r^{1/2}$, (2) $\left|\int_a^b e^{if(x)}dx\right| \leqq \nu/r^{1/2}$. The author has previously determined the best value μ_0 of μ satisfying (1). It is shown that the best value ν_0 of ν satisfying (2) is given by $\mu_0 = \nu_0$. A direct determination of ν_0 is given which, in view of $\nu_0 = \mu_0$, provides an alternative, and more intuitive, determination of μ_0. (Received October 28, 1937.)

404. Dr. H. J. Hamilton: *Some theorems on subsequences.*

Let $\mathfrak{A}$ be the class of all sequences $\{a_k\}$ (finite or infinite) of complex numbers for which $0 < \sum |a_k| < \infty$. The following theorems are proved. (A) Given any sequence $\{a_k\} \in \mathfrak{A}$, there exists a subsequence $\{a_j^*\}$ for which $|\sum a_j^*| = \sup |\sum a'_j|$ over all subsequences $\{a'_j\}$ of $\{a_k\}$. Define $\rho(\{a_k\}) = |\sum a_j^*| / \sum |a_k|$. Then (B) $\rho(\{a_k\}) > 1/\pi$, and (C) $1/\pi = \inf \rho(\{a_k\})$ over all $\{a_k\} \in \mathfrak{A}$. Incidentally, a proof is given that the conditions $\sum |c_k| < \infty$, ψ_k is real (all k), and $\sum |c_k \cos(\phi - \psi_k)| = 0$ for all real ϕ imply that $c_k = 0$ (all k). Somewhat analogous theorems are proved for sequences $\{b_k\}$ for which $\sum |b_k| = \infty$. (Received October 22, 1937.)

405. Mr. R. P. Dilworth. *Abstract residuation over lattices.*

If A and B are ideals in a commutative ring with unit element, the residual of B with respect to A is an ideal $A:B$ having the properties $A \supset (A:B)B$; if $A \supset XB$, then $A:B \supset X$. The purpose of this paper is to give a postulational treatment of a residual defined over a lattice. Since in general no multiplication exists in a lattice, the properties which are proved for the residual are those which are independent of multiplication. Equality is taken as an undefined relation with cross-cut, union, and residual as undefined connections. As an application of the postulates, the residual is determined for a Boolean algebra. (Received October 26, 1937.)

406. Mr. N. A. Hall: *A formal expansion theory for functions defined by two variable power series.*

A generalization to several variables of the Blissard umbral notation for power series enables us to give the expansion of two variable functions in very general sequences of functions, the coefficients in the expansion being given explicitly in terms of the coefficients of the power series for the original function and the coefficients of the power series for the sequence of functions. In particular we give the expansions in products of Bessel functions, $J_n(x) \cdot J_m(y)$, in products of orthogonal polynomials, in mixed products such as Bessel functions and Sonine polynomials, and in a great variety of hypergeometric functions of two variables. (Received October 26, 1937.)

407. Mr. J. W. Green: *A property of harmonic functions in three variables.*

In the plane, a function harmonic within a circle and with a normal deriva-

tive which vanishes on an open set on the circle is uniquely extensible across the open set to every point exterior to the circle. In three dimensions the analagous result does not hold, and an example is exhibited to prove this fact. (Received October 29, 1937.)

408. Mr. D. W. Hall and Dr. G. E. Schweigert: *Properties of invariant sets under pointwise periodic homeomorphisms.*

Let M be a compact separable metric space and T a pointwise periodic homeomorphism of M into itself. Let a point group of M under T be defined as the finite subset of M consisting of a point x of M and all of its images under T. The following results are obtained: (1) Every closed subset L of M which is invariant under T is either vacuous, connected, or has the property that for every possible representation of L as the sum of two mutually separated sets A and B, both A and B are invariant under the same finite power of T. (2) If K is the limit of any convergent sequence of point groups of M under T, and if K contains at least one fixed point under T, then K is a connected set. (Received October 29, 1937.)

409. Professor A. A. Shaw: *A new fragment of Euclid's Elements.* Preliminary report.

The library of the University of Pavia, Italy, has a fragment of Euclid's *Elements*, Book I, in ancient Armenian, which was published, without comment, in "Pazmaveb," vol. 42 (1884), Venice, Italy. The manuscript begins with Definition 19 and ends with Proposition III, and has no date. When compared with the two standard editions of Heiberg and Simson, certain omissions and additions are discovered. The nature of these variations seems to indicate either (a) that the translator used an edition of Euclid *different* from Heiberg and Theonine recensions, or (b) that he used a pre-euclidean "Elements." Furthermore, close study of the style and diction leads the present writer to believe that the translation may be as early as the fifth century. (Received October 29, 1937.)

410. Professor Glenn James: *On the first case of Fermat's Last Theorem.*

In a previous paper limits were established for the parameters x, y, z, of the Fermat equation, $x^n+y^n=z^n$, beneath which this equation can have no solution. For the cases $n=3$ and $n=14{,}000$ these limits were $21{,}147+$ and $10^{112,200}+$, respectively ($14{,}000$ is the limit for n beneath which the equation is known to have no solution). In the present paper these limits are raised to $26{,}855+$ and $(10/7)\,10^{112,200}$, the general limit being raised from $n(2n^2+1)^n$ to $(1/(2^{1/n}-1)-1)$ times that limit. (Received October 29, 1937.)

411. Professor E. T. Bell: *Iterated exponential numbers.*

This twofold infinity of positive integers $\xi_n^{(m)}$, $(m, n=0, 1, \cdots)$, is defined by $G_0(x)=e^x$, $G_{m+1}(x)=\exp\,[G_m(x)-1]$, $G_m(x)=\sum_{n=0}^{\infty}\xi_n^{(m)}x^n/n!$. The $\xi_n^{(1)}$ have been investigated by J. Touchard and the writer. The numbers $\xi_n^{(m)}$ have many remarkable arithmetical properties, including a double periodicity with respect

to a prime modulus p. A specimen of the congruences is $\xi_n^{(m)} \equiv (m+r, m) \bmod p$, where $n = p^r$, and $(\ ,\ )$ is a binomial coefficient. The numbers are closely connected with the Stirling numbers of both kinds, but their arithmetical properties are most readily obtained otherwise. For $m = 1$, the numbers are known in combinatorial analysis and in the theory of partitions; for $m > 1$ they do not seem to have been considered before. (Received October 30, 1937.)

412. Mr. H. A. Arnold: *Differentiation in linear, semi-ordered spaces.*

Properties of the Fréchet and the Gateaux differentials are studied relative to abstract semi-ordered spaces as postulated and treated by Kantorovitch. Implicit function theorems are studied from the purely topological standpoint and also later by the aid of the properties of differentials. Methods are indicated for attacking the problems of the existence theorems of differential equations in abstract semi-ordered spaces. (Received October 30, 1937.)

413. Professor A. D. Michal: *General Riemannian spaces with abstract coordinates and constant curvature.*

In this paper, a definition of "Riemannian" curvature is given for the author's general Riemannian differential geometry with Banach coordinates. In case the Banach space is a finite dimensional arithmetic space the definition is equivalent to the one of classical Riemannian geometry. Schur's well known theorem on spaces of constant Riemannian curvature is generalized. Conformally flat and projectively flat general Riemannian spaces with Banach coordinates are then studied and their theory is used in the study of general Riemannian spaces with Banach coordinates and constant "Riemannian" curvature. (Received October 30, 1937.)

414. Dr. A. H. Diamond: *On the elliptic quintic curve in space of four dimensions.*

The author obtains certain properties of the elliptic quintic curve in space of four dimensions by considering correspondences of points on the curve. Through any real point P of the curve pass four primes which osculate the curve at four real points. Quadruplets of points are thus determined on the curve such that the osculating primes at the points of a quadruplet intersect at a point on the curve. A linear construction of the curve is given by means of three such quadruplets. These twelve points determine four groups of sixteen trisecant planes. The curve is shown to be the intersection of four cubic ruled surfaces which consist of the four systems of secants of the curve which intersect the four groups of trisecant planes. (Received November 1, 1937.)

415. Professor Max Zorn: *Topological studies in the theory of analytic functions.* Preliminary report.

It is shown that many essential features of the theory of analytic functions of one complex variable are derivable from a small set of postulates. These postulates are strictly topological, and it is conjectured that they imply the

complete theory of analytic functions on Riemann surfaces. (Received November 1, 1937.)

416. Professor Morgan Ward: *Structure residuation.*

Given any two elements A and B of an arbitrary structure Σ, the author defines the residual of B with respect to A relative to cross-cut as an element $R=A:B$ of Σ with the properties $A \supset [A:B, B]$; $A \supset [X, B]$ implies $A:B \supset X$. Every structure closed with respect to such an operation or its dual must be distributive, and any distributive structure with a chain condition is closed with respect to residuation. If $B \supset A$, the quotient A/B equals the residual $A:B$. The results of a paper, to appear in December in the Duke Mathematical Journal, on residuation in structures over which a multiplication is defined all apply to the present case on identifying multiplication with cross-cut. In particular the two "distributive laws" $[A, B]:C=[A:C, B:C]$, $A:[B, C] = [A:B, A:C]$ always hold. Interesting results ensue if the additional distributive law $A:[B, C]=[A:B, A:C]$ holds. For a structure with descending chain condition, the powers of any irreducible element form an ordered structure. If we define the "kernel" of any element of the structure as the cross-cut of the highest powers of all irreducible elements dividing it, then all elements with the same kernel form an ordered structure. These facts allow an extension of the fundamental theorem of arithmetic to such structures. Every finite structure closed with respect to residuation is isomorphic to a set of positive integers closed under the operations of L. C. M. and G. C. D. Application is made of this result to studying the free distributive structure based on a finite number of elements. (Received November 1, 1937.)

417. Professor P. H. Daus: *Collineations and central projections.*

This paper discusses the collineation between two planes determined by four pairs of corresponding elements. It proves that the collineation may be established by means of three central projections and gives two different geometric constructions for obtaining such a chain of central projections. (Received October 20, 1937.)

418. Dr. R. M. Robinson: *Note on convex regions on the sphere.*

If G is any convex region on the sphere, and G_0 the diametrically opposite region, then there is on the sphere a circle of radius arctan $(3^{1/2}/2)$, whose interior is either entirely in G or entirely exterior to G and G_0. There is not always a larger circle with this property. (Received October 28, 1937.)

419. Professor E. T. Bell: *Lagrange and Wilson theorems for the generalized Stirling numbers.*

The generalized Stirling number $S_r^{(m)}(n-1)$ is the coefficient of x^{n-r-1} in $\prod_{a=1}^{n-1}(x+a^m)$. A complete discussion of the congruence properties of these numbers with respect to a prime modulus is given. The case $m=1$ is Lagrange's identical congruence; the case $m=2$ was disposed of by Glaisher, whose method, different from the present one, would be difficult to extend to $m>2$. (Received October 30, 1937.)

420. Professor E. T. Bell: *Notes on denumerants.*

If $n, a_1, \cdots, a_r$ are given integers, $n \geqq 0$, $a_1 > 0, \cdots, a_r > 0$, the number $A(n; a_1, \cdots, a_r)$ of sets of non-negative integer solutions of $a_1x_1 + \cdots + a_rx_r = n$ is the denumerant $A(n)$. The characteristic functional equation of $A(n)$ is derived and used to generalize and simplify theorems on denumerants due to Fergola and Sardi. Also a reciprocity between any denumerant $A(n)$ as above and $A(n; b_1, \cdots, b_s)$ is exhibited. This reciprocity also characterizes denumerants. Finally the partition sums of Jacobi, Sylvester, Catalan, and others are generalized, and the generalizations are connected with recurring series. (Received October 30, 1937.)

421. Mr. H. A. Arnold: *Riemannian tensor analysis in abstract spaces.*

A linear form whose values are covariant vectors is taken as a fundamental metric. It is required to be self-adjoint with respect to the contraction operation. The effect of regarding this linear form as the interspace product of an object $g(x)$ in one Banach space with an object $\xi(x)$ in another Banach space is studied with a view to obtaining an abstraction and generalization of the properties of tensors and vectors. (Received October 30, 1937.)

422. Professor A. D. Michal: *Infinitely dimensional Riemannian differential geometries with constant curvature.*

In this paper a special study is made of those geometries introduced in a previous paper (abstract 43-11-413) for which the Banach space is taken to be either a space of continuous functions or a Hilbert space. (Received October 30, 1937.)

423. Dr. Erich Rothe: *Topological proofs of uniqueness theorems in the theory of differential and integral equations.*

Let $y = f(x)$ be a uniform continuous representation defined for all points x of a sphere V of the n-dimensional euclidean space. Then the "order of a point y with respect to the image of the boundary of V" is a well known topological concept; if the equation $y_0 = f(x)$ has but one solution in V, the order equals ± 1. The converse is true only under additional conditions, for instance if the "indices" of the presumable solutions x all have the same sign. It is shown that for a certain class of representations the same consideration holds in Banach space and may be used for the proof of uniqueness theorems concerning differential and integral equations. Proofs of uniqueness theorems based on other topological ideas were first given by R. Caccioppoli (1931) and later on by G. Scorza (1935, 1936). (Received November 1, 1937.)

424. Mr. Philip Hartman: *Mean motions of almost periodic functions and the Riemann zeta function.*

It is proved that every uniformly almost periodic function with linearly independent exponents has a mean motion; a result recently proved (Hartman, van Kampen and Wintner, *Mean motions and distribution functions*, American Journal of Mathematics, vol. 59 (1937), pp. 261–269) only with the exclusion

of certain limiting cases and by using the ergodic theorem of Birkhoff. The present general treatment is based on an application of the Jessen function of analytic almost periodic functions (Jessen, *Über die Nullstellen einer analytischen fastperiodischen Funktion*, Mathematische Annalen, vol. 108 (1933), pp. 485–516) and on the other hand, the direct estimate of Fourier transforms in case of convolutions (Wintner, *Upon a statistical method in the theory of diophantine approximations*, American Journal of Mathematics, vol. 55 (1933), pp. 309–331). The method can be extended so as to apply, without the assumption of Riemann's hypothesis, both for $\zeta(s)$ and $\log \zeta(s)$ for every fixed $\sigma > 1/2$. (Received October 30, 1937.)

425. Dr. Ruth R. Struik: *The place of Euclid's tenth book in ancient and modern science.*

The tenth book of Euclid, edited in a popular edition by Enriques and the author of this paper, is presented as a unique masterpiece of research of Euclid's time compared to the other twelve books (except perhaps the fifth), which are compendium schoolbooks rather than original treatises. The whole of the thirteen books are considered a "cours d'analyse" of an "academicien" of more than 2,000 years ago, teaching at one of the many primitive colleges of Plato's and Archimedes' time. (Received October 25, 1937.)

426. Dr. Ruth R. Struik: *A problem connected with the "axiomatic of affine geometry."*

The problem is: What is the form of an infinitesimal operator in the sense of Lie which is, microcosmically, an analogy to the affine reflection? The affine reflexion is the transformation of the plane so that all points are mirrored on a straight line s, the mirror being in the direction of a given vector p. It is similar to a reflexion in ordinary euclidean geometry, only the rays are not orthogonal to s, but make an arbitrary angle with s. (Received October 25, 1937.)

427. Dr. O. E. Lancaster: *Some results concerning the behavior at infinity of real continuous solutions of algebraic difference equations.*

This paper treats of the rate of increase of real continuous solutions of algebraic difference equations; that is, of equations of the form $f[x, y(x), y(x+1), \cdots, y(x+m)] = 0$, where f is a polynomial in its arguments x, $y(x)$, $y(x+1), \cdots, y(x+m)$ with rational coefficients. The following results are obtained: First, a solution of a first order algebraic difference equation cannot equal or exceed $e_2[xl_n(x)]$ for all $x > x_0(n)$, where $e_2(x)$ denotes the second iterate of the exponential, $l_n(x)$ denotes the nth iterate of the logarithm, and n is any integer. Second, $ce_2[xl_n(x)]$ cannot be a solution of any algebraic difference equation. Third, if $y(x)$ is a solution of an algebraic difference equation of the mth order, and if the increase of $y(x)$ satisfies certain very restrictive regularity conditions, then $|y(x)| < e_2[xl_n(x)]$ for $x > x_0(n)$. Fourth, unless regularity conditions are imposed upon the solutions there is no upper bound for the rate of increase of the solutions of all algebraic difference equations of higher than

first order. This last result is a direct analogue of the results obtained by Vijayaraghavan for algebraic difference equations (Comptes Rendus, Paris, vol. 194 (1932), pp. 827–829). (Received November 2, 1937.)

428. Professor E. G. Olds: *A moment-generating function which is useful in solving certain matching problems.*

A given target deck consists of n/s different sets of s cards, those in each set being alike. A second deck, identical with the given deck, is thoroughly mixed and the order compared with that of the target deck. This paper is concerned with the moments of the theoretical distribution of numbers of correct matchings. A moment-generating function is derived and the values of the first four moments are obtained. The third and fourth moments are, respectively, $[s(n-s)(n-2s)]/[(n-1)(n-2)]$ and $\{s(n-s)[(n-2s)(n-3s)(3s+1)+(s-1)(12ns-n-18s^2-6s)]\}/[(n-1)(n-2)(n-3)]$ (except for certain trivial cases when $n \leqq 3$). The values of the first two moments, obtained in another way, were communicated by Greenwood (abstract 43-9-331). Results for the special case $s=1$ were given by Chapman (American Journal of Psychology, vol. 46, p. 294). (Received October 26, 1937.)

429. Professor V. C. Poor: *The Cauchy singular integral and circulation functions.*

In this paper the principal value of the singular integral of Cauchy is expressed in explicit form; so also are the principal values of this integral. Necessary and sufficient conditions are obtained for a function restricted by a Lipschitz condition to be a *circulation function* in a region bounded by a simple Jordan curve. Similar results are obtained for the exterior region. (Received October 29, 1937.)

430. Professor V. C. Poor: *The analog to the Cauchy integral and circulation functions of the second kind.*

The analog to the Cauchy integral is found and studied. Incidentally the analog to the Pompeiu theorem is deduced. It is then shown how a circulation function of the second kind may be determined for a region from its boundary values. Both the interior and exterior cases are considered. (Received October 29, 1937.)

431. Dr. W. E. Sewell: *Note on the Faber coefficients of a continuous function.*

Let C be an analytic Jordan curve in the z-plane. Let $f(z)$ be analytic in C, continuous in $\overline{C}$, and let $f^{(p)}(z)$ have a modulus of continuity $\omega(\delta)$ on C. Let $\sum_{\nu=0}^{\infty} a_\nu P_\nu(z)$ be the development of $f(z)$ in the Faber polynomials belonging to C. Then we have $|a_\nu| \leqq (Mr^p/\nu^p)\omega(\epsilon/\nu)$ where M and ϵ are constants independent of ν, and $1/r$ is the capacity of the set bounded by C. (Received November 1, 1937.)

432. Professor F. C. Smith: *Interrelations among the funda-*

mental solutions of the generalized hypergeometric equation when $p=q+1$. I. *Non-logarithmic cases.*

In this paper, the relations between the non-logarithmic solutions of the generalized hypergeometric equation about the point $z=0$ and those about the point $z=\infty$ are developed for the case in which $p=q+1$. The results generalize those of Mehlenbacher (see abstract 42-5-169) who developed the relations between the fundamental solutions of the hypergeometric equation, and extend those of Barnes (Proceedings of the London Mathematical Society (2), vol. 5 (1907), pp. 59–116) who obtained the asymptotic developments of the solutions of the generalized hypergeometric equation for those cases in which $p<q+1$. (Received October 23, 1937.)

433. Dr. Olga Taussky: *Analytical methods in hypercomplex systems.*

Let S be a hypercomplex system with respect to the field of real numbers. The set Λ of all elements λ such that $\lim_{n=\infty}\lambda^n=0$ throws light on the algebraic structure of the ring in many ways (see also the author's paper in Compositio Mathematica, vol. 3 (1936)). Among other results the following may be of interest: The necessary and sufficient condition that for any two elements λ, λ' of Λ the product $\lambda\lambda'$ also is contained in Λ is that the residue class ring S/R (R the radical of S) is the direct sum of division algebras F_1, F_2, $\cdots$ and $F_iF_k=0$ if $i\neq k$. (Received October 23, 1937.)

434. Mr. W. H. Ingram: *Kirchhoff constraints and linear graphs.*

In a connected system of line segments (network) representing a constrained dynamical system, the author introduces topologically (and requires one vertex to be) a *ground-point* or *ground*; also *Lagrangean connection* between each free vertex and ground; and asserts that all currents (velocities) are circuital (traversing the Lagrangean connections in part). Maxwell's topological axiom for unconstrained systems is obtained as a theorem. External forces e_i on the dynamical system are associated with corresponding line segments but associable with the *chords* of any tree by an application of Blakesley's n-furcation theorem. The cyclomatic number is given by the number of segments minus the number of Lagrangean connections. Potentials associated with the vertices are computed from the Lagrangean multipliers associated with the Lagrangean connections. (Received November 1, 1937.)

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PART II. APPLIED MATHEMATICS

BARTELS (J.). See LUBBERGER (F.).

BECKER (R.). See LUBBERGER (F.).

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——— Systèmes de référence et mouvements (physique classique). Part 3: Mécanique Newtonienne et gravitation. (Actualités Scientifiques et Industrielles, No. 481.) Paris, Hermann, 1937. 144 pp.

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——— Systèmes de référence et mouvements (physique relativiste). Part 1: Genèse des théories de la relativité. (Actualités Scientifiques et Industrielles, No. 486.) Paris, Hermann, 1937. 46 pp.

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WORTHING (A. G.). See BLACKWOOD (O. H.).

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

IOWA CITY, IOWA, November 26–27, 1937.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than October 30; they will appear in the November issue of the Bulletin. By invitation of the Program Committee, addresses will be delivered by Professor W. J. Trjitzinsky on *Singular point problems in the theory of linear differential equations*, and by Professor A. H. Copeland on *Fundamental concepts of the theory of probability*.

PASADENA, CALIFORNIA, November 27, 1937.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than October 30; they will appear in the November issue of the Bulletin.

INDIANAPOLIS, INDIANA, ANNUAL MEETING, December 28–30, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than November 23, 1937. Abstracts received by the Secretaries by November 2 will appear in the November issue of this Bulletin. The Annual Gibbs Lecture will be delivered by Professor Charles A. Kraus, on *The present status of the theory of electrolytes*, before a joint session of this Society and the American Association for the Advancement of Science. Professor G. C. Evans will deliver his retiring address as Chairman of Section A of the latter organization at a joint meeting of Section A with this Society and the Mathematical Association of America. His subject will be *Mathematical progress in theoretical economics*. There will also be a joint session of this Society with the Association for Symbolic Logic, and one with the Institute of Mathematical Statistics. By invitation of the Program Committee, Professor Alonzo Church will deliver an address entitled *The constructive second number class*.

NEW YORK CITY, February 26, 1938.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than January 28. Abstracts received on or before December 31, 1937, will appear in the January issue of the Bulletin. By invitation of the Program Committee, Professor P. A. Smith will give an address entitled *The topology of groups of transformation*.

R. G. D. RICHARDSON, *Secretary of the Society*.

Articles for insertion in the BULLETIN should be addressed to the Managing Editor, D. R. CURTISS, Northwestern University, Evanston, Ill. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450–459 Ahnaip St., Menasha, Wis., or Low Memorial Library, 531 West 116th St., New York, N. Y.

The initial fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1936, p. 67) are payable to the Treasurer of the Society, Professor P. A. Smith, Low Memorial Library, 531 West 116th St., New York, N. Y.

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